String breaking on a lattice

P. Pennanen

Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
E-mail: petrus@nordita.dk

String breaking is a non-perturbative long-distance feature of QCD that is involved in for example meson decays. A mixing analysis of lattice operators at zero temperature gives the level splitting and mixing energy between the broken and unbroken string states.

1 Background

A major motivation for studying string breaking in QCD has been its nature as a fundamental feature, sometimes even taught to high-school students, that hadn't been reproduced from the theory. A phenomenological motivation is its significant role in decays of e.g. mesons. The problems in a theoretical reproduction have on the analytical side been the non-perturbative nature of string breaking, while on the non-perturbative side the long distance range has hindered lattice methods.

An analogue for $q\bar{q}$ creation in a chromoelectric QCD flux tube is the creation of real e^+e^- in a constant electric QED field. The probability of the latter was found to be $\propto \exp -\pi m^2/|eE|$ by Schwinger [1]. A similar factor of $\exp -4m^2/b_s$ for a flux tube with string tension b_s has been calculated in strong coupling QCD [2]. In other words, the virtual particle and anti-particle have to separate by tunneling a distance inversely proportional to the force experienced in the (chromo)electric field in order to balance out the energy needed for their masses. In vacuum the $q\bar{q}$ are produced with $J^{PC} = 0^{++}$, which is known as the Quark Pair Creation model [3]. For creation in a flux tube, however, only the component of J parallel to the tube is a good quantum number.

The quark pair creation model has been combined with a flux tube model to calculate decay widths of hybrid mesons [4,5]. Here the flux tube is taken to be a string of coupled quantum mechanical harmonic oscillators vibrating in transverse planes, which predicts level orderings well but flux tube shapes poorly [6]. With an L-dependent $q\bar{q}$ creation vertex a selection rule suppressing decays of low-lying hybrids to identical mesons was obtained [5], which is relevant to our lattice studies.

2 Lattice operators

The $Q\bar{Q}$ ground state potential $V_0(R)$ crosses twice the energy of a $Q\bar{q}$ meson at approximately 1.2 fm in both quenched and unquenched lattice QCD. The parameters of these simulations can be found in Ref. [7]. Even though one might expect the Wilson loop measuring $V_0(R)$ to level out on unquenched lattices this hasn't been seen despite extensive efforts (for a review see [8]). The reason seems to be a poor overlap of the Wilson loop with the $Q\bar{q}+\bar{Q}q$ state which gets significant only at temperatures close to the deconfinement transition (see [8,10] for references).

A variational approach, where the operators representing both the $Q\bar{Q}$ and $Q\bar{q}+\bar{Q}q$ states are explicitly present, has shown much more promise with Higgs and adjoint colour source calculations. Here a matrix $C(R,T)$ of the correlations between these operators is diagonalised according to

$$C(R,T)v_\alpha(T) = \lambda_\alpha(T)C(R,T-1)v_\alpha(T). \qquad (1)$$

This gives eigenvalues $\lambda = \exp -E(R)$ and eigenvectors v for each state α.

In the simplest case of a 2×2 matrix the eigenenergies $E_\alpha(R)$ can be obtained by diagonalising

$$\begin{pmatrix} V_0(R) & x(R) \\ x(R) & M_0(R) \end{pmatrix}, \qquad (2)$$

where $M_0(R)$ is the ground state energy of the meson-antimeson system and $x(R)$ is its mixing energy with the $Q\bar{Q}$ state. At the string breaking point $R = R_b$ the potential $V_0(R) = M_0(R)$ and the splitting between $E_{0,1}$ is $2x$.

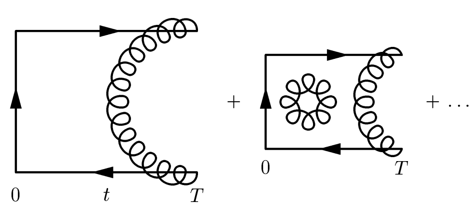

Figure 1: Illustration of Eq. 3.

The observables of the variational approach are also very hard to measure on the lattice, as the gap x is small. Even with the use of all-to-all propagator estimates [9] that give better statistics than conventional techniques it was hard to get a proper signal. Fortunately we found a way to measure x explicitly by extracting it from the lattice correlators.

As illustrated in Fig. 1, the correlation between $Q\bar{Q}$ and $Q\bar{q}+\bar{Q}q$ states can be pictured as propagation of the two-quark state from time 0 until time t, where $q\bar{q}$ are created with mixing energy x and propagate until time T. In the unquenched theory additional terms with more mixings come from vacuum bubbles appearing inside the diagram. This can be written as

$$U(T) = x(R) \sum_{t=0}^{T} \sum_{k=0}^{\infty} w_k e^{-V_k(r)t} \sum_{l=0}^{\infty} e^{-M_l(r)(T-t)} d_l \; [+O(x^3)]_{\mathrm{unquenched}}. \quad (3)$$

The connected correlator B of the meson-antimeson state with itself can be written in the same manner. When the Wilson loop $W(T) = \sum_{k=0}^{\infty} w_k^2 e^{-V_k(r)T}$ and the unconnected correlator $D(T) = \sum_{l=0}^{\infty} d_l^2 e^{-M_l(r)T}$ are measured separately the B and U provide two different ways of extracting the mixing x [10].

We obtained a first result for its value in unquenched SU(3) lattice QCD at the string breaking point: $x/a = 46(8)$ MeV. Its small value enables the use of only the first term in Eq. 3 and the corresponding equation for box, and makes it difficult to observe the mixing directly from the spectrum. From the Schwinger factor we expect x to be roughly constant for $R \approx R_b$ due to a stable field density in a long flux tube. The same factor suggests our estimate to be maybe 20% lower than the physical one due to a higher quark mass.

3 An "order parameter" for string breaking?

There has been speculation based on an instanton calculation [11] that an adjoint string in pure SU(3) would not break at all – this calculation has recently been heavily criticized [12]. Some people are also looking for an order parameter for fundamental string breaking in full QCD that would distinguish between quenched and unquenched lattice configurations (the measurement programs for both are identical). The potential $V_0(R)$ measured with Wilson loops should flatten only for unquenched, but this hasn't been found to actually happen as discussed above. The variational approach has been critized for giving a flattening ground state potential also in the quenched model.

In the quenched case the variational matrix $C(R,T)$ turns out to be inconsistent; $U \neq 0$, allowing the extraction of x relevant for the full theory, but no mixing appears in the energies of the transfer matrix. There is no energy gap between ground and first excited state for quenched. This inconsistency is a reflection of the non-unitary nature of the quenched approximation when light quark degrees of freedom are explicitly introduced in the correlators. Reflection positivity is lost and there is no QFT left. Thus we don't think there is a need for an "order parameter". However, such a parameter can be obtained by e.g. measuring the $Q\bar{Q}$ and $Q\bar{q} + q\bar{Q}$ energies separately and looking for a gap between them at the string breaking point.

4 Excited strings

In the breaking of a first excited state of the flux tube conservation of its angular momentum forces one of the resulting mesons to have $L > 0$ as in the selection rule mentioned above. In practise de-excitation into a $Q\bar{Q} + q\bar{q}$ state seems to be a much more relevant decay channel [13]. In this and higher-lying cases the energies and mixing between hybrid $Q\bar{Q}$, two heavy-light $Q\bar{q} + q\bar{Q}$ and $Q\bar{Q}+q\bar{q}$ states can be studied on the lattice with our techniques, including a new method for calculating disconnected diagrams [14].

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